MMAT 5010 Linear Analysis (2024-25): Homework 10
Deadline: 25 Apr 2025

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $T : \ell^2 \to \ell^2$ be the right operator, that is $T(x_1, x_2, ...) := (0, x_1, x_2, , ...)$ for $(x_1, x_2, ...) \in \ell^2$. Find T^*, T^*T and TT^*.

2. Let X be a Hilbert space and let $T, S \in L(X)$. Show that

   (a) $(T + S)^* = T^* + S^*$ and $(\alpha T)^* = \overline{\alpha} T^*$ for $\alpha \in \mathbb{C}$.

   (b) $(TS)^* = S^* T^*$.

   (c) if T is invertible, that in $T^{-1} \in L(X)$ exists, then $(T^{-1})^* = (T^*)^{-1}$.

$* * *$ **End** $* * *$